INSTITUTO TECNOLÓGICO DA AERONAUTICA

VESTIBULAR - 98

PROVA DE MATEMATICA

GABARITO

01.

- como $y = \text{sen}\,2x$ e $y = \cos 2x$ têm período principal $\frac{2\pi}{2} = \pi$, então $\underline{f}$ tem período π
- como $y = \text{sen}\,2x$ é ímpar e $y = \cos 2x$ é par, então $\underline{f}$ não é par nem ímpar

→ (C)

02.

$$tg^{10}x - 5tg^{8}x.\sec^{2}x + 10tg^{6}x.\sec^{4}x - 10tg^{4}x.\sec^{6}x + 5tg^{2}x.\sec^{8}x - \sec^{10}x =$$
$$= (tg^{2}x - \sec^{2}x)^{5} = (-1)^{5} = -1$$

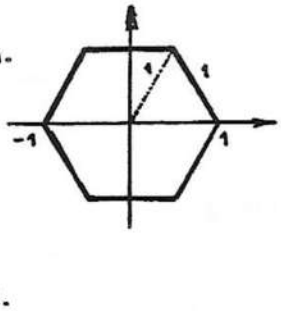
→ (D)

03.

- $A = M^{-1}.B.M \rightarrow \det(A) = \cancel{\det}(M^{-1}).\det(B).\cancel{\det}(M) \rightarrow \det(A) = \det(B)$
- como A é 2×2, $\det(-A^{t}) = \det(A^{t}).(-1)^{2} = \det(A^{t}) = \det(A) = \det(B)$

→ (A)

04.



$$S = 6.\frac{1^{2}\sqrt{3}}{4} \rightarrow S = \frac{3\sqrt{3}}{2}$$

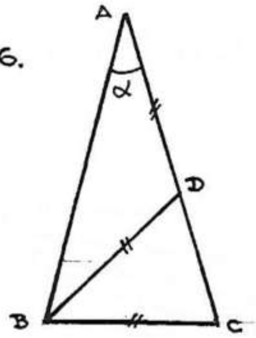
→ (D)

05.

$$z = x + yi \rightarrow z^{3} = x^{3} + 3x^{2}yi - 3xy^{2} - y^{3}i = (x^{3} - 3xy^{2}) + (3x^{2}y - y^{3})i \rightarrow z^{3} = 1 + i$$
$$|z^{3}|^{2} = 2 \rightarrow |z|^{6} = 2 \rightarrow |z| = \sqrt[6]{2}$$

→ (B)

06.



- ΔADB é isósceles $\rightarrow A\hat{B}D = B\hat{A}D = \alpha$

  $\rightarrow B\hat{D}C = 2\alpha$
- ΔBCD é isósceles $\rightarrow B\hat{C}D = B\hat{D}C = 2\alpha$
- ΔABC é isósceles $\rightarrow A\hat{B}C = B\hat{C}A = 2\alpha$

  $\rightarrow \alpha + 2\alpha + 2\alpha = 180° \rightarrow 5\alpha = 180° \rightarrow \alpha = 36°$ //

$\rightarrow$ (C)

07.

$$\lim_{n \to \infty} S_n = \frac{a_1}{1-q} = 3a_1 \rightarrow 1-q = \frac{1}{3} \rightarrow q = \frac{2}{3} = a_1$$

$$S_3 = \frac{2}{3} \cdot \frac{1-\left(\frac{2}{3}\right)^3}{1-\frac{2}{3}} = \frac{2}{3} \cdot \frac{1-\frac{8}{27}}{\frac{1}{3}} = \frac{38}{27}$$ //

$\rightarrow$ (E)

08.

$$2\log_y 7 = \log_{y^2} 7 + \log_{2y} 7 \rightarrow \frac{2}{\log_7 y} = \frac{1}{\log_7 y^2} + \frac{1}{\log_7 2y} \rightarrow \frac{2}{\log_7 y} = \frac{1}{2\log_7 y} + \frac{1}{\log_7 2y}$$

$$\rightarrow \frac{3}{2\log_7 y} = \frac{1}{\log_7 2y} \rightarrow 3\log_7 2y = 2\log_7 y \rightarrow \log_7 (2y)^3 = \log_7 y^2$$

$$\rightarrow 8y^3 = y^2, \text{ como } y \neq 0 \rightarrow 8y = 1 \rightarrow y = \frac{1}{8}$$ //

$\rightarrow$ (D)

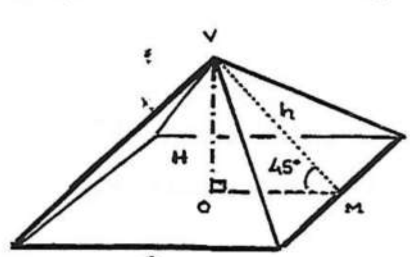
09.

- total de anagramas: 12!
- anagramas com as 5 vogais juntas: (8!)(5!)

$\rightarrow n = 12! - (8!)(5!)$ //

$\rightarrow$ (C)

10.



$OM = 1 \rightarrow h = VM = \sqrt{2}$

$S_b = 2^2 = 4$

$S_\ell = 4 \cdot \frac{1}{2} 2 \cdot \sqrt{2} = 2\sqrt{2}$

$\rightarrow \frac{S_b}{S_\ell} = \frac{1}{\sqrt{2}} = \frac{\sqrt{2}}{2}$ //

$\rightarrow$ (D)

11.

(I) FALSA, pois $f(x) \cdot f(y) = (-3a^x) \cdot (-3a^y) = 9a^{x+y} \neq -3a^{x+y} = f(x+y)$

(II) FALSA, pois $a^x > 0 \rightarrow -3a^x < 0 \rightarrow Im_f = \mathbb{R}_-^* \neq \mathbb{R}$

(III) VERDADEIRA, pois como $0 < a < 1 \rightarrow x_1 > x_2 \rightarrow a^{x_1} < a^{x_2} \rightarrow -3a^{x_1} > -3a^{x_2}$ e $f(0) = -3$

e $\lim_{x \to \infty} f(x) = 0$

$\rightarrow$ (E)

12.

$(f\circ g)(x) = f(g(x)) \rightarrow (g(x))^2 - 9 = x - 6 \rightarrow (g(x))^2 = x + 3 \geq 0 \rightarrow x \geq -3$ //

$\rightarrow$ (A)

13.

. P.A.: $x_1, x_2, x_3 \rightarrow 2x_2 = x_1 + x_3 \rightarrow x_1 + x_2 + x_3 = 3x_2 = 0 \rightarrow x_2 = 0$

· fazendo-se $y = e^x$, temos: $y_1 \cdot y_2 \cdot y_3 = e^{x_1} \cdot e^{x_2} \cdot e^{x_3} = e^{x_1 + x_2 + x_3} = e^0 = 1$

$$\left.\begin{array}{l} \rightarrow -\frac{b}{2} = 1 \rightarrow b = -2 \\ y_2 = e^0 = 1 \rightarrow 2 + a + 7 - 2 = 0 \rightarrow a = -7 \end{array}\right| \rightarrow a - b = -5$$

$\rightarrow$ (D)

14.

$\rightarrow$ recíproca de 2ª espécie e grau par $\rightarrow$ raízes ± 1

	1	2	a	0	$-a$	-2	-1
1	1	3	$a+3$	$a+3$	3	1	0
-1	1	2	$a+1$	2	1	0	

$\rightarrow x^4 + 2x^3 + (a+1)x^2 + 2x + 1 = 0 \quad (\div x^2) \rightarrow \left(x^2 + \frac{1}{x^2}\right) + 2\left(x + \frac{1}{x}\right) + a + 1 = 0$

· $y = x + \frac{1}{x} \rightarrow x^2 + \frac{1}{x^2} = y^2 - 2 \rightarrow y^2 + 2y + a - 1 = 0$

<u>CONDIÇÕES</u>: . $\Delta \geq 0 \rightarrow 4 - 4(a-1) \geq 0 \rightarrow a \leq 2$

· as raízes, em módulo, devem ser maiores ou iguais a 2, como a soma é -2, uma deve ser menor ou igual a -2 a outra maior ou igual a 2

$f(2) \leq 0 \rightarrow 4 + 4 + a - 1 \leq 0 \rightarrow a \leq -7$

$f(-2) \leq 0 \rightarrow 4 - 4 + a - 1 \leq 0 \rightarrow a \leq 1$

$\rightarrow$ (C)

15.

$h(x)$ é do 2º grau $\rightarrow p(x) = q(x)(x-2) + 26 = (ax^2 + bx + c)(x^2 + x - 1) + 8x - 5$

$$\left.\begin{array}{l} \cdot\ x = 2 \rightarrow 26 = (4a + 2b + c)(4 + 2 - 1) + 11 \rightarrow 4a + 2b + c = 3 \\ \cdot\ x = 0 \rightarrow 13 \times (-2) + 26 = c \cdot (-1) - 5 \rightarrow c = -5 \\ \cdot\ x = 1 \rightarrow 26 \cdot (-1) + 26 = (a + b + c) \cdot (1 + 1 - 1) + 3 \rightarrow a + b + c = -3 \end{array}\right\} \rightarrow \begin{cases} a = 2 \\ b = 0 \end{cases}$$

$\rightarrow h(x) = 2x^2 - 5 \rightarrow h(2) + h(3) = 2 \times 4 - 5 + 2 \times 9 - 5 = 16$ //

$\rightarrow$ (A)

16.

$$\delta_3^{(1)} = \begin{vmatrix} 1 & 1 & 0 \\ 1 & -3 & 1 \\ 0 & -2 & a \end{vmatrix} = 0 \rightarrow -3a - (-2+a) = 0 \rightarrow a = \frac{1}{2}$$

$$\Delta_3^{(2)} = \begin{vmatrix} 1 & -1 & 0 \\ 1 & 2 & -1 \\ 2 & -b & 3 \end{vmatrix} = 0 \rightarrow 6 + 2 - (b-3) = 0 \rightarrow b = 11$$

$$\rightarrow \frac{b}{a} = 22$$

$\rightarrow$ (B)

17.

$\det(A) = 2 \;;\; \det(B) = 2 \rightarrow \det(AB) = 4$

$C = AB \rightarrow c_{11} = 2 + a + a^2$ e $c_{22} = 3 + a$

$D = C^{-1} \rightarrow d_{11} + d_{22} = \frac{1}{\det(AB)} \cdot (c_{11} + c_{22}) = \frac{1}{4}(5 + 2a + a^2)$

$\rightarrow$ (E)

18.

- $x+3 \geq 1 \rightarrow x \geq -2 \rightarrow \log_5(x+3) \geq 0$
  $\rightarrow 4x \geq -(x^2+3) \rightarrow x^2 + 4x + 3 \geq 0 \rightarrow x \leq -3$ ou $x \geq -1$ $\quad \rightarrow S_1 = [-1, \infty)$
- $0 < x+3 \leq 1 \rightarrow -3 < x \leq -2 \rightarrow \log_5(x+3) \leq 0$
  $\rightarrow 4x \leq -(x^2+3) \rightarrow x^2 + 4x + 3 \leq 0 \rightarrow -3 < x \leq -1$ $\quad \rightarrow S_2 = (-3, -2]$

$\rightarrow S = S_1 \cup S_2 = (-3, -2] \cup [-1, \infty)$

$\rightarrow$ (A)

19.

$$\sqrt{3}.\mathrm{tg}x - \sqrt{3}.\frac{2\mathrm{tg}x}{1+\mathrm{tg}^2x} + \frac{1-\mathrm{tg}^2x}{1+\mathrm{tg}^2x} = 0 \rightarrow \sqrt{3}\mathrm{tg}x + \sqrt{3}\mathrm{tg}^3x - 2\sqrt{3}\mathrm{tg}x + 1 - \mathrm{tg}^2x = 0$$

$$\rightarrow \sqrt{3}\mathrm{tg}^3x - \mathrm{tg}^2x - \sqrt{3}\mathrm{tg}x + 1 = 0 \rightarrow \sqrt{3}\mathrm{tg}x(\mathrm{tg}^2x - 1) - (\mathrm{tg}^2x - 1) = 0$$

$$\rightarrow (\mathrm{tg}^2x - 1)(\sqrt{3}\mathrm{tg}x - 1) = 0 \rightarrow \mathrm{tg}x = \pm 1 \rightarrow x \in \left\{\frac{\pi}{4}, \frac{3\pi}{4}, \frac{5\pi}{4}, \frac{7\pi}{4}\right\}$$

$$\mathrm{tg}x = \frac{\sqrt{3}}{3} \rightarrow x \in \left\{\frac{\pi}{6}, \frac{7\pi}{6}\right\}$$

$$\rightarrow \text{SOMA} = \frac{16\pi}{3}$$

$\rightarrow$ (B)

20.

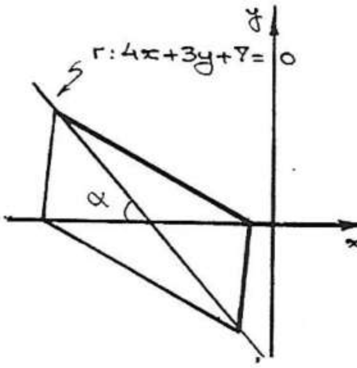
(I) VERDADEIRA, pois $\frac{\cancel{n}(n-3)}{2} = \cancel{n} \rightarrow n = 5$

(II) FALSA, pois $\frac{\cancel{n}(n-3)}{2} = 4\cancel{n} \rightarrow n = 11$

(III) VERDADEIRA, pois $\frac{n_d}{n} = \frac{n-3}{2} \in \mathbb{N} \rightarrow n-3$ é par $\rightarrow n$ é ímpar

$\rightarrow$ (B)

21.



$$tg\,\alpha = -m_r = \frac{4}{3}$$
$$\rightarrow \operatorname{sen}\alpha = \frac{4}{5}$$
$$S = \frac{1}{2} d_1 \cdot d_2 \operatorname{sen}\alpha = \frac{1}{2} 4 \cdot 6 \cdot \frac{4}{5} = \frac{48}{5}$$

→ (E)

22.

- Poliedro original: $V + F = A + 2 \rightarrow n + m = 18$

$$32 = 3f_3 + 4f_4 \quad ; \quad f_4 + f_3 = m$$

- Novo poliedro: $2A' = 4(f_4+1) \rightarrow A' = 2f_4 + 2$ $\quad\Big|\rightarrow n - 1 + f_4 + 1 = 2f_4 + 2 + 2$

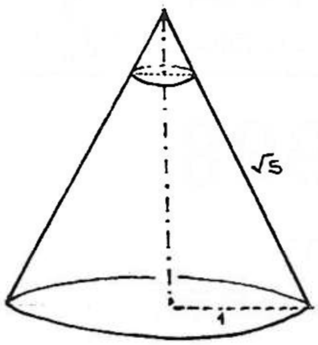
$V' = n - 1$

$$\rightarrow n = f_4 + 4$$

$$\rightarrow \begin{cases} 3f_3 + 4f_4 = 32 \\ f_4 + 4 + f_4 + f_3 = 18 \rightarrow f_3 + 2f_4 = 14 \end{cases} \rightarrow f_3 = 4 \rightarrow f_4 = 5 \rightarrow \begin{matrix} m = 9 \\ n = 9 \end{matrix}$$

→ (B)

23.



$$h = 2 \quad ; \quad \frac{V_{n+1}}{V_1} = 2 \rightarrow V_{n+1} = 2V_1 = \frac{1}{3}\pi\, 1^2 \cdot 2 \rightarrow V_1 = \frac{\pi}{3}$$

$$S_n = \frac{(V_1 + V_{n+1})(n+1)}{2} = 2\pi \rightarrow \left(\frac{\pi}{3} + \frac{2\pi}{3}\right)(n+1) = 4\pi$$
$$\rightarrow n = 3$$

→ V_T = razão da P.A.

$$\frac{2\pi}{3} = \frac{\pi}{3} + 3 \cdot V_T \rightarrow V_T = \frac{\pi}{9}$$

→ (C)

24.

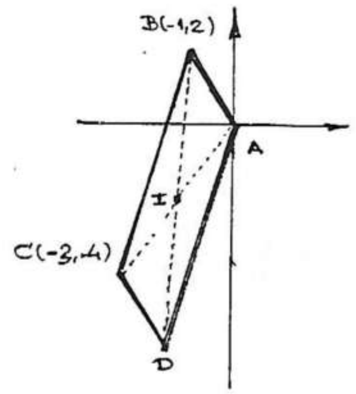
$$H: \frac{(y-2)^2}{5} - \frac{(x+3)^2}{4} = 1 \rightarrow \text{CENTRO } (-3,2)$$
$$c^2 = a^2 + b^2 = 9 \rightarrow c = 3 \quad \Big| \rightarrow \text{Focos: } \begin{cases} (-3,5) \\ (-3,-1) \end{cases}$$

$$T: (y-3)^2 = 4(x-1) \rightarrow \text{VÉRTICE: } (1,3)$$

$$\rightarrow (x+3)^2 + (y-5)^2 + (x+3)^2 + (y+1)^2 = 3\left((x-1)^2 + (y-3)^2\right)$$
$$\rightarrow 2x^2 + 12x + 18 + 2y^2 - 8y + 26 = 3x^2 - 6x + 3 + 3y^2 - 18y + 27$$
$$\rightarrow x^2 + y^2 - 18x - 10y - 14 = 0 \rightarrow (x-9)^2 + (y-5)^2 = 120$$

→ (E)

25.



I é ponto médio de BD e de AC:

$$\left.\begin{array}{l} x_D + (-1) = 0 + (-3) \rightarrow x_D = -2 \\ y_D + 2 = 0 + (-4) \rightarrow y_D = -6 \end{array}\right| \rightarrow D(-2,-6),$$

$$\left.\begin{array}{l} m_{AB} = -2 \\ m_{AD} = 3 \end{array}\right| \rightarrow \operatorname{tg} A = \frac{2+3}{1-2\times 3} = -1 \rightarrow \hat{A} = \frac{3\pi}{4},$$

$$\rightarrow \hat{B} = \frac{\pi}{4}, \rightarrow \text{(D)}$$