ITA Matemática 1991 – Professor Zé Ricardo

GABARITO - 90/91

MATEMÁTICA

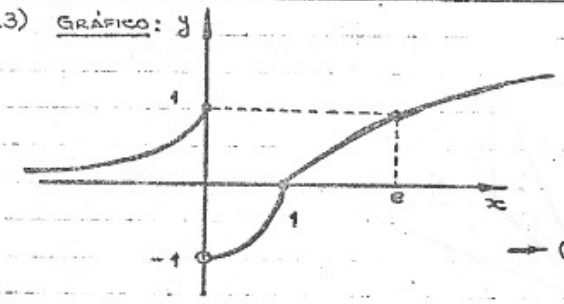
(1) I. $g\circ f(-x) = g(f(-x)) = g(f(x)) = g\circ f(x)$
→ $g\circ f$ é PAR (V)

II. $f\circ g(-x) = f(g(-x)) = f(-g(x)) = f(g(x)) = f\circ g(x)$ → $f\circ g$ é PAR (V)

III. $y = f(x) \to x = f^{-1}(\,)$
$\to -y = f(-x) \to -x = f^{-1}(-y)$
→ f^{-1} é ÍMPAR (V)
→ (E)

(2) $y = \frac{a^x - a^{-x}}{2} \to a^{2x} - 2y\cdot a^x - 1 = 0$
$\to a^x = \frac{2y \pm \sqrt{4y^2+4}}{2} > 0$
$\to a^x = y + \sqrt{y^2+1} \to x = \log_a(y+\sqrt{y^2+1})$
→ (C)

(3) GRÁFICO: y



→ (B)

(4) $z = x+yi$, $w = a+bi$ → $wz = ax - by + (bx+ay)i$

$wz + \overline{wz} = 2\cdot Re(wz) = 2(ax-by)$
$\to 2ax - 2by + c = 0$
$\to m = a/b$ → (D)

(5) $1+z = 1+\cos t + i\,\text{sen}\,t = 2\cos^2\frac{t}{2} + i\,2\,\text{sen}\frac{t}{2}\cos\frac{t}{2} =$
$= 2\cos\frac{t}{2}\left(\cos\frac{t}{2} + i\,\text{sen}\frac{t}{2}\right) = 2\cos\frac{t}{2}\cdot\text{cis}\frac{t}{2}$

$1-z = 1-\cos t - i\,\text{sen}\,t = 2\,\text{sen}^2\frac{t}{2} - i\,2\,\text{sen}\frac{t}{2}\cos\frac{t}{2} =$
$= 2\,\text{sen}\frac{t}{2}\left(\text{sen}\frac{t}{2} - i\cos\frac{t}{2}\right) = -2i\,\text{sen}\frac{t}{2}\cdot\text{cis}\frac{t}{2}$

$\to \frac{1+z}{1-z} = -\frac{1}{i}\cdot\text{ctg}\frac{t}{2} = i\cdot\text{ctg}\frac{t}{2}$ → (A)

(6) $x_1 + x_2 = 1$, $x_1+x_2+x_3 = 6$ → $x_3 = 5$

$\to P(5) = 0 \to 125 - 150 - 5m^2 + 30 = 0$
$\to 5m^2 = 5 \to m = \pm 1$ → (C)

(7) $P(x) = 12x^3 - 16x^2 - 3x + 4$

$P(-2) = -96 - 64 + 6 + 4 < 0$
$P(-1) = -12 - 16 + 3 + 4 < 0$ → $x_1 \in (-1,0)$
$P(0) = 4 > 0$ → $x_2 \in (0,1)$
$P(1) = 96 - 64 - 3 + 4 < 0$ → $x_3 \in (1,2)$
$P(2) = 96 - 64 - 6 + 4 > 0$

→ (A)

(8) I. $3x^4 - 10x^3 + 10x^2 - 3 = (x^2-1)((x^2+03) - 10x) =$
$= (x^2-1)(3x^2 - 10x + 3) = 0$
$\to x = \pm 1$; $x = 3$; $x = 1/3$ → (V)

II. → (F)

III. → $S_1' = 2S_1$; $S_2' = 4S_2$; $S_3' = 8S_3$ → (V)
→ (B)

(9) $\cdot\ x^2 + x + 1 > 0$; $\forall x \in \mathbb{R}$

$\cdot$ $x \le -3$ ou $x \ge 1$: $\to x^2 + 2x - 3 \ge 0$
$\to x^2 + x + 1 \le x^2 + 2x - 3$
$\to x \ge 4$ → $S_1 = [4, \infty)$

$\cdot$ $-3 < x < 1$: $\to x^2 + 2x - 3 < 0$
$\to x^2 + x + 1 \le -x^2 - 2x + 3$
$\to 2x^2 + 3x - 2 \le 0 \to -2 \le x \le 1/2$
$\to S_2 = [-2, 1/2]$

$\cdot$ $S = S_1 \cup S_2$ → (A)

(10) $P(x) = (x-1)\cdot Q(x) - 6$

$\to a_3 = b_4$
$2 = b_3 - b_4 = b_4(q-1)$
$a_4 = b_2 - b_3$
$8 = b_1 - b_2 = b_4(q^3 - q^2)$ $\xrightarrow{(\div)}$ $q^2 = 4 \to q = 2$
$-32 = b_0 - b_1$ → $b_4 = 2$
$a_3 = -6 - b_0$ → $b_3 = 4$
→ $b_2 = 8$
→ $a_4 = 4$
$\to b_4 + a_4 = 6$ → (B)

(11) $\ln a_1 \cdot \ln q = 24$
$\ln a_2 + \ln a_3 = 26 \to \ln a_1 + \ln q + \ln a_1 + 2\ln q = 26$
$\to 2\ln a_1 + 3\ln q = 26$
$\to 2\cdot\frac{24}{\ln q} + 3\ln q = 26 \to 3\ln^2 q - 26\ln q + 48 = 0$
$\to \ln q = 6$ ou $\ln q = 8/3$
$\to \ln a_1 = 4$
$\to a_1 = e^4$; $q = e^6$ → $a_n = e^4\cdot(e^6)^{n-1} = e^{6n-2}$ → (A)

(12) $\log x + \log(2x+3) \le \log 2$
$\to \log x\cdot(2x+3) \le \log 2 \to x(2x+3) \le 2$
e $x > 0$

$\to 2x^2 + 3x - 2 \le 0 \to -2 \le x \le 1/2$
$\to 0 < x \le 1/2$ → (C)

(13) $A = (3+1)^n = 4^n$
$B = (11+1)^{n-1} = 12^{n-1}$
$\to \ln B - \ln A = (n-1)\ln 12 - n\ln 4 = \ln 6561 - \ln 4$
$\to n\ln 4 - \ln 4 + (n-1)\ln 3 = n\ln 4 = 8\ln 3 - \ln 4$
$\to (n-1)\ln 3 = 8\ln 3 \to n = 9$ → (E)

ITA Matemática 1991 – Professor Zé Ricardo

GABARITO - 90/91

(14) HIPÓTESES:

MAT	FIS	QUIM	OUT
5	3	2	2
5	3	1	3
5	3	0	4
5	2	2	3
5	2	1	4

$\rightarrow$

$N = C_7^5 \times C_3^3 \times C_4^2 \times C_2^2 + C_7^5 \times C_3^3 \times C_4^1 \times C_2^3 + C_7^5 \times C_3^3 \times C_4^0 \times C_1^4 + C_8^5 \times C_3^2 \times C_4^1 \times C_2^3 + C_7^5 \times C_3^2 \times C_4^1 \times C_2^1 =$

$= 756 + 336 + 21 + 1512 + 252 = 2877 \rightarrow (D)$

(15)

$mA + nB = \begin{bmatrix} 2m-n & m+n \\ 3m & 5m+n \end{bmatrix}$

$\det(mA+nB) = (2m-n)(5m+n) - (m+n)3m =$

$= 10m^2 - 3mn - n^2 - 3m^2 - 3mn = 7m^2 - 6mn - n^2$

$7m^2 - 6mn - n^2 = 0 \rightarrow n^2 + 6mn - 7m^2 = 0$

$(n-m)(n+7m) = 0$

$n \neq m \rightarrow n = -7m \rightarrow (C)$

(16) $B = M - M^{-1} = M - M^t$

$B^t = (M - M^t)^t = M^t - M = -B \rightarrow (D)$

(17)

$\Delta = \begin{vmatrix} 1 & 0 & 1 & 1 \\ 1 & k & 0 & k^2 \\ 1 & 0 & k+1 & 1 \\ 1 & 0 & 1 & k \end{vmatrix} = k \cdot \begin{vmatrix} 1 & 1 & 1 \\ 1 & k+1 & 1 \\ 1 & 1 & k \end{vmatrix} =$

$= k \cdot \begin{vmatrix} 1 & 0 & 0 \\ 1 & k & 0 \\ 1 & 0 & k-1 \end{vmatrix} = k^2(k-1) \neq 0 \rightarrow k \neq 0 \text{ e } k \neq 1$

$\rightarrow (D)$

(18)

$x - y + 2z - t = 0$

$3x + y + 3z + t = 0 \quad (+)$

$x - y - z - 5t = 0$

$\rightarrow 2x + z + 2t = 0$

$-3z - 4t = 0$

$t = 3a \rightarrow z = -4a \rightarrow x = -a \rightarrow y = -12a$

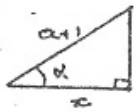
$\rightarrow x + y + z + t = -14a \rightarrow (C)$

(19) $\operatorname{sen} A = \frac{a}{2R} = \frac{2\sqrt{3}}{2 \cdot 2\sqrt{3}} = \frac{1}{2} \rightarrow \hat{A} = 30° \text{ ou } \hat{A} = 150°$

$3\hat{B} = \hat{A} + \hat{C} + \hat{B} = 180° \rightarrow \hat{B} = 60° \rightarrow \hat{C} = 90°$

$\rightarrow c = 2R = 4\sqrt{3} \rightarrow (A)$

(20) $\alpha = \operatorname{arc\,sen} \frac{a-1}{a+1} \in Q_I \rightarrow \frac{a-1}{a+1} > 0 \rightarrow a > 1$



$x^2 = (a+1)^2 - (a-1)^2 = 4a$

$\rightarrow x = 2\sqrt{a}$

$\rightarrow \operatorname{tg}\alpha = \frac{a-1}{2\sqrt{a}}$

$\rightarrow \operatorname{tg}(\alpha+\beta) = \frac{\frac{a-1}{2\sqrt{a}} + \frac{1}{2\sqrt{a}}}{1 - \frac{a-1}{4a}} = \frac{a/2\sqrt{a}}{3a+1/4a} = \frac{2a\sqrt{a}}{3a+1}$

$\rightarrow (C)$

(21) $\cos^3 x + (a-1)\cos^2 x - (a+b)\cos x + b = 0 \rightarrow$

$\rightarrow \cos^2 x(\cos x - 1) + a\cos x(\cos x - 1) - b(\cos x - 1) = 0$

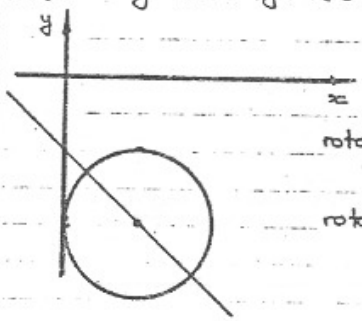
$\rightarrow (\cos x - 1)(\cos^2 x + a\cos x - b) = 0$

- $\cos x - 1 = 0 \rightarrow x = 0$
- $\cos^2 x + a\cos x - b = 0$

$x = \pi/2 \rightarrow f(0) = -b < 0$

$x = 0 \rightarrow f(1) = 1 + a - b > 0 \rightarrow b < a + 1 \rightarrow (B)$

(22) $x^2 + y^2 - 2x + 4y + 4 \leq 0 \rightarrow (x-1)^2 + (y+2)^2 \leq 1$

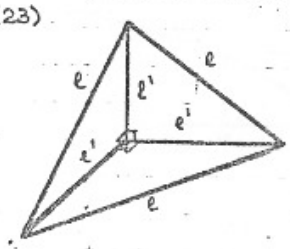


rotação de $\pi \rightarrow V = \frac{4}{3}\pi R^3 = \frac{4}{3}\pi$

rotação de $\frac{\pi}{3} \rightarrow V' = \frac{1}{3}V$

$\rightarrow V' = \frac{4\pi}{9} \rightarrow (D)$

(23)



$\rightarrow \ell' = \frac{\ell\sqrt{2}}{2}$

$\rightarrow V = \frac{1}{3} \cdot \frac{\ell'^2}{2} \cdot \ell' = \frac{1}{6}\ell'^3 =$

$= \frac{1}{6} \cdot \frac{\ell^3 \cdot 2\sqrt{2}}{8} = \frac{\ell^3\sqrt{2}}{24}$

$\rightarrow (E)$

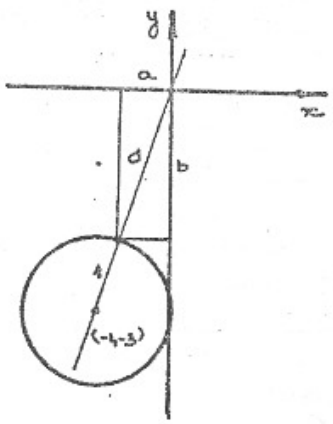
(24) · ponto média de MN: $P(2,-4)$

$m_{MN} = \frac{-6+2}{-4-8} = \frac{-4}{-12} = \frac{1}{3} \rightarrow m = -3$

$m: y + 4 = -3(x-2) \rightarrow m: 3x + y - 2 = 0$

$R = d_{(0,m)} = \frac{|-2|}{\sqrt{9+1}} = \frac{\sqrt{10}}{5} \rightarrow (D)$

(25) $x^2 + y^2 + 2x + 6y + 9 = 0 \rightarrow (x+1)^2 + (y+3)^2 = 1$



$d = \sqrt{1+9} - 1 = \sqrt{10} - 1$

$\frac{a}{-1} = \frac{\sqrt{10}-1}{\sqrt{10}} = 1 - \frac{\sqrt{10}}{10}$

$\rightarrow a = \frac{\sqrt{10}}{10} - 1$

$b = 3a$

$\rightarrow (C)$