GABARITO - 89/90

MATEMÁTICA

(1) $f(x) = \dfrac{1+e^x}{1-e^x} \rightarrow f(-x) = \dfrac{1+e^{-x}}{1-e^{-x}} \times \dfrac{e^x}{e^x} = \dfrac{e^x+1}{e^x-1} =$

$= -f(x) \rightarrow$ FUNÇÃO ÍMPAR

$g(x) = x.\text{sen}\,x \rightarrow g(-x) = -x.\text{sen}(-x) = -x.(-\text{sen}\,x)$

$= x.\text{sen}\,x = g(x)$

$\rightarrow$ FUNÇÃO PAR $\rightarrow$ (C)

(2) (I) • $f(2) = f(3) = 4 \rightarrow$ NÃO INJETORA

• $f^{-1}([3,5]) = f^{-1}(\{4\}) = (1,\infty) \rightarrow$ FALSA

(II) • $\nexists x$ tq $f(x) = 10 \rightarrow$ NÃO SOBREJETORA

• $f^{-1}([3,5]) = f^{-1}([2,6]) = (1,\infty) \rightarrow$ VERDADEIRA

(III) • FALSA (f é NÃO INJETORA)

$\rightarrow$ (C)

(3) $f(x) = \dfrac{2x-3}{x-2} + 1 = y \leftrightarrow f^{-1}(y) = x$

$\rightarrow \dfrac{2x-3}{x-2} = y-1 \rightarrow 2x-3 = (y-1)x - 2y + 2$

$\rightarrow (y-1-2)x = 2y-3-2$

$\rightarrow x = \dfrac{2y-5}{y-3}$; $y \neq 3 \rightarrow$ (E)

(4) • $z^3 = i = \text{cis}\dfrac{\pi}{2} \rightarrow z = \text{cis}\left(\dfrac{2k\pi}{3} + \dfrac{\pi}{6}\right)$; $k = 0,1,2$

$\rightarrow S_1 = \left\{\text{cis}\dfrac{\pi}{6}; \text{cis}\dfrac{5\pi}{6}; \text{cis}\dfrac{3\pi}{2}\right\} =$

$= \left\{\dfrac{\sqrt{3}}{2} + \dfrac{1}{2}i; -\dfrac{\sqrt{3}}{2} + \dfrac{1}{2}i, -i\right\}$

• $z^2 + (2+i)z + 2i = 0$

$\rightarrow S_2 = \{-2, -i\}$

$\rightarrow S_1 \cap S_2 = \{-i\} \rightarrow$ (D)

(5) $z = x + yi \rightarrow z + 1 = (x+1) + yi$

$1 + |z| = |1+z| \rightarrow 1 + \sqrt{x^2+y^2} = \sqrt{(x+1)^2+y^2}$

$\rightarrow \cancel{1} + 2\sqrt{x^2+y^2} + \cancel{x^2} + \cancel{y^2} = \cancel{x^2} + 2x + \cancel{1} + \cancel{y^2}$

$\rightarrow \cancel{2}\sqrt{x^2+y^2} = \cancel{2}x \rightarrow x \geq 0 \rightarrow \text{Re}\,z \geq 0$

$\rightarrow \cancel{x^2} + y^2 = \cancel{x^2} \rightarrow y = 0$

$\rightarrow \text{Im}\,z = 0 \rightarrow$ (B)

(6) $p(x)$ do 5º grau $\rightarrow$ nº ímpar de raízes reais

coef. reais $\rightarrow$ nº par de raízes complexas

$\rightarrow$ 3 RAÍZES REAIS: $x_1 \cdot a$; x_1; $\dfrac{x_1}{a}$

$\therefore \cancel{a}x_1 \cdot x_1 \cdot \dfrac{x_1}{\cancel{a}} = x_1^3 = \dfrac{1}{64} \rightarrow x_1 = \dfrac{1}{4}$

• $(a+\cancel{b}i) + (a-\cancel{b}i) + \dfrac{7}{8} = \dfrac{78}{16} = \dfrac{39}{8} \rightarrow 2a = 4 \rightarrow a = 2$

• $(a+bi)(a-bi) \cdot \dfrac{1}{64} = \dfrac{5}{16} \rightarrow a^2 + b^2 = 20 \rightarrow b = \pm 4$

• $\dfrac{\alpha}{16} = x_1 \cdot x_2 \cdot x_3 \cdot x_4 \cdot x_5 \left(\dfrac{1}{x_1} + \dfrac{1}{x_2} + \dfrac{1}{x_3} + \dfrac{1}{x_4} + \dfrac{1}{x_5}\right)$

$\rightarrow \dfrac{\alpha}{16} = \dfrac{5}{16}\left(4 + 2 + 8 + \dfrac{1}{2+4i} + \dfrac{1}{2-4i}\right)$

$\rightarrow \alpha = 70 + \dfrac{5(2-\cancel{4i}) + 5(2+\cancel{4i})}{4+16} = 71 \rightarrow$ (C)

(7) $|\ln(\text{sen}^2 x)| = \ln(\text{sen}^2 x) \rightarrow \ln(\text{sen}^2 x) \geq 0$

$\rightarrow \text{sen}^2 x \geq 1 \rightarrow \text{sen}^2 x = 1$

$\rightarrow \text{sen}\,x = \pm 1 \rightarrow x = k\pi + \dfrac{\pi}{2}$ $(k \in \mathbb{Z}) \rightarrow$ (A)

(8) sendo $3^x = y \rightarrow 12y^3 - 19y^2 + 8y - 1 = 0$

$x = \dfrac{1}{3}$ é RAIZ $\rightarrow$

	12	-19	8	-1
$1/3$	12	-15	3	0

$\rightarrow 12y^2 - 15y + 3 = 0 \rightarrow 4y^2 - 5y + 1 = 0$

$\rightarrow y_2 = 1$ e $y_3 = 1/4$

$\rightarrow 3^{x_1} = \dfrac{1}{3} \rightarrow x_1 = -1$

$3^{x_2} = 1 \rightarrow x_2 = 0$

$3^{x_3} = \dfrac{1}{4} \rightarrow x_3 = -\log_3 4$

$\rightarrow \Sigma x_i = -1 - \log_3 4 =$

$= -\log_3 3 - \log_3 4 =$

$= -\log_3 12 \rightarrow$ (A)

(9) PG: a_1; $a_1 e^{-2a}$, $a_1 e^{-4a}$

$\rightarrow a_1(1 + e^{-2a} + e^{-4a}) = 7$

$\rightarrow a_1(e^{-4a} - 1) = 3$

$\rightarrow \dfrac{e^{-4a} + e^{-2a} + 1}{e^{-4a} - 1} = \dfrac{7}{3}$

$\rightarrow 3.e^{-4a} + 3.e^{-2a} + 3 = 7e^{-4a} - 7$

$e^{-2a} = x \rightarrow 4x^2 - 3x - 10 = 0$

$x = 2$ ou $x = \cancel{-5/4}$

$\rightarrow e^{-2a} = 2 \rightarrow -2a = \ln 2 \rightarrow a = -\ln\sqrt{2} \rightarrow$ (D)

(10) • $g(x) < 1 \rightarrow \dfrac{2x-3}{x-1} < 1 \rightarrow \dfrac{2x-3}{x-1} - 1 < 0$

$\rightarrow \dfrac{x-2}{x-1} < 0 \rightarrow 1 < x < 2$

• $g(x) > -1 \rightarrow \dfrac{2x-3}{x-1} > -1 \rightarrow \dfrac{2x-3}{x-1} + 1 > 0$

$\rightarrow \dfrac{3x-4}{x-1} > 0 \rightarrow x < 1$ ou $x > 4/3$

$\rightarrow |g(x)| < 1 \rightarrow 4/3 < x < 2 \rightarrow f \circ g(x) = 1$

$\rightarrow$ (C)

(11) $T_{k+1} = C_8^k \cdot (\cos\alpha . x)^{8-k} \cdot \left(\text{sen}\,\alpha . \dfrac{1}{x}\right)^k =$

$= C_8^k \cdot \cos^{8-k}\alpha . \text{sen}^k\alpha . x^{8-2k}$

$x^0 \rightarrow 8 - 2k = 0 \rightarrow k = 4$

$T_5 = C_8^4 \cdot \cos^4\alpha . \text{sen}^4\alpha = \dfrac{35}{8} \rightarrow 70 \cdot \dfrac{1}{16} \cdot \text{sen}^4 2\alpha = \dfrac{35}{8}$

$\rightarrow \text{sen}^4 2\alpha = 1 \rightarrow 2\alpha = \dfrac{\pi}{2} \rightarrow \alpha = \dfrac{\pi}{4} \rightarrow$ (D)

$0 < 2\alpha < \pi$

(12) $x = a^2.\text{tg}\,t + 1 \rightarrow \text{tg}\,t = \dfrac{x-1}{a^2} \geq 0 \rightarrow x \geq 1$

$y^2 = b^2\sec^2 t - b^2 \rightarrow \sec^2 t = \dfrac{y^2+b^2}{b^2} = \dfrac{y^2}{b^2} + 1$

$\rightarrow \dfrac{y^2}{b^2} + \cancel{1} = \cancel{1} + \dfrac{(x-1)^2}{a^4} \rightarrow y^2 = \dfrac{b^2}{a^4}(x-1)^2$

$\rightarrow y = \pm\dfrac{b}{a^2}(x-1) \rightarrow$ (D)

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(13) $2\,\text{sen}(\theta-60^\circ) = \cos(\theta+60^\circ)$

$\rightarrow 2(\text{sen}\theta\cdot\underbrace{\cos 60^\circ}_{1/2} - \underbrace{\text{sen}\,60^\circ}_{\sqrt{3}/2}\cdot\cos\theta) = \cos\theta\cdot\underbrace{\cos 60^\circ}_{1/2} - \text{sen}\theta\,\underbrace{\text{sen}\,60^\circ}_{\sqrt{3}/2}$

$\rightarrow (2+\sqrt{3})\,\text{sen}\theta = (1+2\sqrt{3})\cos\theta$

$\rightarrow \text{tg}\theta = \frac{1+2\sqrt{3}}{2+\sqrt{3}}\times\frac{2-\sqrt{3}}{2-\sqrt{3}} = -4+3\sqrt{3} \rightarrow a=-4,\ b=3$

$\rightarrow$ (B)

(14) $\det A = \cancel{2}\,\text{sen}^2 x - \cancel{2}\log_3 10 = 2(\text{sen}^2 x - \log_3 10)$

como $\log_3 10 > 0 \rightarrow \det A < 0;\ \forall x \in \mathbb{R}$

$\rightarrow$ (C)

(15) • $\det A \neq 0 \rightarrow \det A^t \neq 0 \rightarrow \det(ABCA) \neq 0$

$\rightarrow \det C \neq 0 \rightarrow C$ é INVERSÍVEL

• $\det A.\det B.\det C.\det \cancel{A} = \det \cancel{A}$

$\rightarrow \det C = \frac{1}{\det A.\det B} = \frac{1}{\det(AB)} = \det(AB)^{-1}$

$\rightarrow$ (A)

(16) (I) $\rightarrow$ FALSA (determinado × indeterminado)

(II) $\rightarrow$ FALSA

(III) $\begin{cases} x+y = 5 \\ y+z = 8 \\ x+y+z = 10 \end{cases} \xrightarrow{(-)} x = 2 \rightarrow y = 3 \rightarrow z = 5$

na 3ª equação: $4\times2-3+2\times5 = 15 \neq 14 \rightarrow$ FALSA

$\rightarrow$ (E)

(17) $\Delta = \begin{vmatrix} a_1 & a_1+1 & \cdots & a_1+n-1 \\ a_2 & a_2+1 & \cdots & a_2+n-1 \\ \vdots & \vdots & & \vdots \\ a_n & a_n+1 & \cdots & a_n+n-1 \end{vmatrix} = \begin{vmatrix} a_1 & 1 & \cdots & n-1 \\ a_2 & 1 & \cdots & n-1 \\ \vdots & \vdots & & \vdots \\ a_n & 1 & & n-1 \end{vmatrix} =$

$= 0 \rightarrow$ (E) (exceto para $n=2$)

(18) $\left.\begin{array}{l} n = 2a+1 \\ a = 2b+1 \\ b = 2c+1 \end{array}\right| \rightarrow a = 4c+3 \rightarrow n = 8c+7$

o 1º recebeu $a+c = 5c+3$

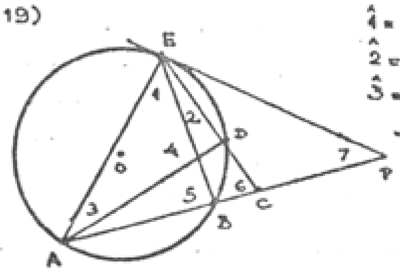
o 2º recebeu $b+c = 3c+1$

$\rightarrow \frac{5c+3}{3c+1} = \frac{29}{17} \rightarrow 85c+51 = 87c+29$

$\rightarrow 2c = 22 \rightarrow c = 11$

$\rightarrow n = 95 \rightarrow$ (B)

(19)



$\hat{1} = 49^\circ \rightarrow \widehat{AB} = 98^\circ$

$\hat{2} = 18^\circ \rightarrow \widehat{BD} = 36^\circ$

$\hat{3} = 34^\circ \rightarrow \widehat{DE} = 68^\circ$

$\rightarrow \widehat{EA} = 158^\circ$

$\hat{4} = \frac{\widehat{BD}+\widehat{EA}}{2} = 97^\circ$

$\hat{5} = \frac{\widehat{EA}}{2} = 79^\circ$

$\hat{6} = \frac{\widehat{EA}-\widehat{BD}}{2} = 61^\circ$

$\hat{7} = \frac{\widehat{EA}-\widehat{BD}}{2} = 27^\circ$

$\rightarrow$ (D)

(20) β: $\frac{|3x-4y+12|}{\sqrt{3^2+(-4)^2}} = \frac{|3x-4y+4|}{\sqrt{3^2+(-4)^2}}$

• $3x-4y+12 = -(3x-4y+4)$

$\rightarrow 3x-4y+8 = 0 \rightarrow$ (A)

(21) $\left.\begin{array}{l} x^2+y^2-6\sqrt{2}y=0 \\ y=\sqrt{2}x \end{array}\right| \rightarrow x^2+2x^2-6.2x = 0$

$\rightarrow 3x^2 = 12x$

$\rightarrow x_1 = 0 \rightarrow y_1 = 0$

$x_2 = 4 \rightarrow y_2 = 4\sqrt{2}$

$\rightarrow \overline{P_1P_2}^2 = 16+16\times2 = 16\times3 \rightarrow \overline{P_1P_2} = 4\sqrt{3}$

$CP_1 = CP_2 = R = 3\sqrt{2}$

$\rightarrow 2p = 6\sqrt{2}+4\sqrt{3} \rightarrow$ (E)

(22) $y_A = 0 \rightarrow A(-7/2, 0)$

$x_B = 0 \rightarrow B(0, 7/3)$ $\rightarrow M = (-\frac{7}{4}, \frac{7}{6})$

$m_r = ⋯\ 3x+2y+k=0 \therefore M \in r \rightarrow -\frac{21}{4}+\frac{7}{3}+k = 0$

$\rightarrow k = 35/12$

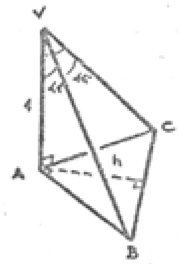
$\rightarrow r: 36x+24y+35 = 0$

$\rightarrow d = \frac{|9+4+35|}{12.\sqrt{9+4}} = \frac{48}{12\sqrt{13}} = \frac{4}{\sqrt{13}} \rightarrow$ (B)

(23) $\frac{h}{2x} = \frac{3}{5} \rightarrow h = \frac{6x}{5}$

$V = \frac{1}{\cancel{3}}\cdot\frac{x^2\sqrt{3}}{\cancel{4}\,2}\cdot\frac{\cancel{6}x}{5} = \frac{x^3\sqrt{3}}{10} \rightarrow$ (D)

(24)



Δ VAB e Δ VAC são retângulos isósceles $\rightarrow AB = AC = 1$

$VB = VC = \sqrt{2}$

Δ VBC é isósceles $\rightarrow$

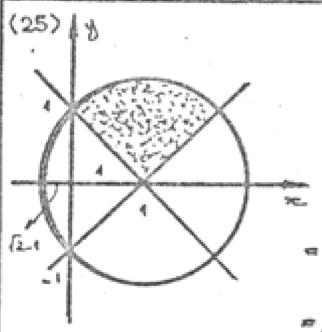
$BC^2 = 2+2-2\sqrt{2}.\sqrt{2}.\frac{\sqrt{2}}{2} = 4-2\sqrt{2}$

$\rightarrow BC = \sqrt{4-2\sqrt{2}}$

Δ ABC é isósceles $\rightarrow h^2 = 1-\frac{4-2\sqrt{2}}{4} = \frac{\sqrt{2}}{2} = \frac{2\sqrt{2}}{4}$

$\rightarrow V = \frac{1}{3}.\frac{1}{2}\sqrt{4-2\sqrt{2}}.\frac{\sqrt{2\sqrt{2}}}{2}.1 = \frac{1}{6}\sqrt{2\sqrt{2}-\sqrt{2}} \rightarrow$ (A)

(25)



$V = V_{esf} - 2V_{cone} - 2V_{seg} =$

$= \frac{4}{3}\pi.(\sqrt{2})^3 - 2.\frac{\pi}{3}.1.1 - 2.\frac{1}{3}\pi(\sqrt{2}-1)^2(3\sqrt{2}-(\sqrt{2}-1))$

$= \frac{8\sqrt{2}\pi}{3} - \frac{2\pi}{3} - \frac{2\pi}{3}(3-2\sqrt{2})(2\sqrt{2}+1)$

$= \frac{8\pi}{3} \rightarrow$ (B)

OBS: $V_{seg} = \frac{1}{3}\pi h^2(3r-h)$